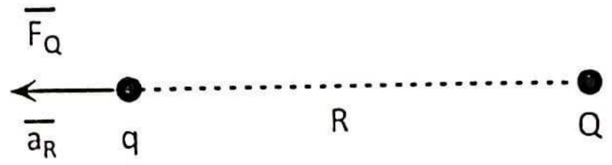
Lec (03)

Chapter 2 : Coulomb Forces and Electric Field Intensity

3.1 Coulomb's Law

There is a force between two charges which is directly proportional to the charge magnitudes and inversely proportional to the square of the separation distance



$$F = \frac{q\,Q}{4\pi\epsilon R^2}$$

F : Magnitude of the force between Q and q (N)

R : The distance between the two charges (m)

Q , q : The magnitudes of the charges (C)

ϵ : permittivity of the medium (F/m or C^2/Nm^2)

$$\epsilon = \epsilon_o \epsilon_r$$

$$\epsilon_o = 8.854 \times 10^{-12} = \frac{10^{-9}}{36\,\pi} \text{ F/m}$$

For free space $\epsilon_r = 1$

The force exerted on q by Q

$$\bar{F}_Q = \frac{q\,Q}{4\pi\epsilon R^2}\,\bar{a}_R$$

$\bar{a}_R$: Unit vector directed from Q to q

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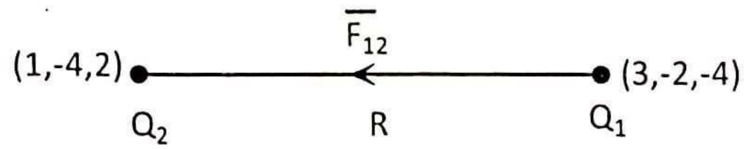
1 A 2 mC positive charge is located in vacuum at $P_1(3,-2,-4)$ and a $5\mu C$ negative charge is at $P_2(1,-4,2)$:

a) Find the vector force on the negative charge.

b) What is the magnitude of the force on the charge at P_1?

Answer



$$\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon R^2}\,\bar{a}_{R_{12}}$$

$$\bar{R}_{12} = -2\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z$$

$$R = |\bar{R}| = \sqrt{4+4+36} = \sqrt{44}\ \text{m}$$

$$\bar{a}_R = \frac{\bar{R}_{12}}{|\bar{R}|} = \frac{-2\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z}{\sqrt{44}}$$

$$\bar{F}_{12} = \frac{2\times10^{-3}\times -5\times10^{-6}}{4\pi\times 8.85\times10^{-12}\times 44}\left(\frac{-2\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z}{\sqrt{44}}\right)$$

$$\bar{F}_{12} = (-2.04)\left(\frac{-2\bar{a}_x - 2\bar{a}_y + 6\bar{a}_z}{\sqrt{44}}\right)$$

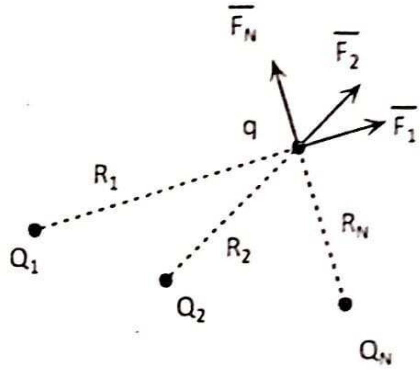
$$\bar{F}_{12} = 0.616\,\bar{a}_x + 0.616\,\bar{a}_y - 1.848\,\bar{a}_z$$

$$|\bar{F}| = \left|\frac{Q_1 Q_2}{4\pi\epsilon R^2}\right| = 2.04\ N$$

Force due to several charges

$$\bar{F}_T = \bar{F}_1 + \bar{F}_2 + \cdots + \bar{F}_N$$

$$\bar{F}_T = \frac{q\,Q_1}{4\pi\epsilon {R_1}^2}\,\bar{a}_{R_1} + \frac{q\,Q_2}{4\pi\epsilon {R_2}^2}\,\bar{a}_{R_2} + \cdots + \frac{q\,Q_N}{4\pi\epsilon {R_N}^2}\,\bar{a}_{R_N}$$



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[2] Find the force on a 100 μC charge at $(0,0,3)m$ if four like charges of $20\mu C$ are located on the x and y axes at $\pm 4m$.

$$\bar{F}_T = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$\bar{F}_T = \frac{Q_1Q}{4\pi\epsilon R^2}\,\bar{a}_{R_1} + \frac{Q_2Q}{4\pi\epsilon R^2}\,\bar{a}_{R_2} + \frac{Q_3Q}{4\pi\epsilon R^2}\,\bar{a}_{R_3} + \frac{Q_4Q}{4\pi\epsilon R^2}\,\bar{a}_{R_4}$$

$$\bar{F}_T = \frac{Q_1Q}{4\pi\epsilon R^2}(\bar{a}_{R_1} + \bar{a}_{R_2} + \bar{a}_{R_3} + \bar{a}_{R_4})$$

$$\bar{R}_1 = -4\bar{a}_y + 3\bar{a}_z$$

$$R = |\bar{R}_1| = \sqrt{16+9} = 5$$

$$\bar{a}_{R_1} = \frac{\bar{R}_1}{|\bar{R}_1|}$$

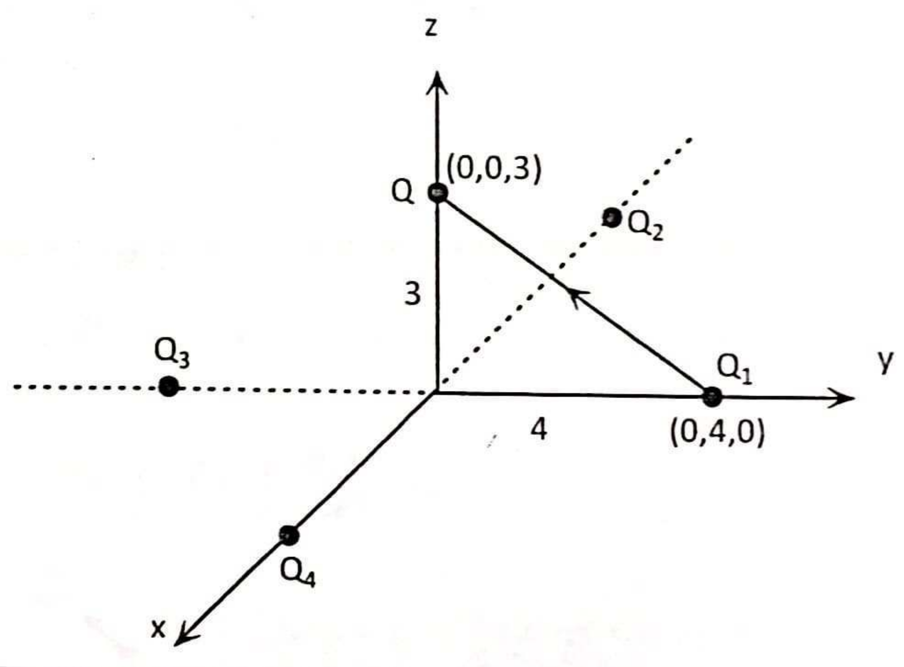
$$\bar{a}_{R_1} = \frac{-4\bar{a}_y + 3\bar{a}_z}{5}$$

$$\bar{a}_{R_1} = -\frac{4}{5}\bar{a}_y + \frac{3}{5}\bar{a}_z$$

$$\bar{a}_{R_3} = \frac{4}{5}\bar{a}_y + \frac{3}{5}\bar{a}_z$$

$$\bar{a}_{R_2} = \frac{4}{5}\bar{a}_x + \frac{3}{5}\bar{a}_z$$

$$\bar{a}_{R_4} = -\frac{4}{5}\bar{a}_x + \frac{3}{5}\bar{a}_z$$



$$\bar{F}_T = \frac{Q_1Q}{4\pi\epsilon R^2}\left(4\left(\frac{3}{5}\right)\bar{a}_z\right)$$

$$\bar{F}_T = \frac{100\times10^{-6}\times 20\times10^{-6}}{4\pi\times 8.85\times10^{-12}\times 25}\left(4\left(\frac{3}{5}\right)\bar{a}_z\right)$$

$$\boldsymbol{\bar{F}_T = 1.73\ \bar{a}_z\ N}$$

3.2 Electric Field Intensity

The electric field intensity $\bar{E}$ due to Q is defined as

"The force per unit charge on the test charge q"

$$\bar{E} = \frac{\bar{F}}{q} \quad \text{N/C } or \text{ V/m}$$

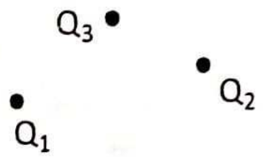
The electric field due to a point charge is

$$\bar{E}_Q = \frac{Q}{4\pi\epsilon R^2}\,\bar{a}_R \quad \text{V/m}$$

3.3 Charge distribution

(1) Point Charges

- Are assumed to exist at isolated points in space as shown in fig



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(2) Line charge distribution

- The charge is distributed over a line with a linear charge density ρ_L (C/m)
- The total charge on the line

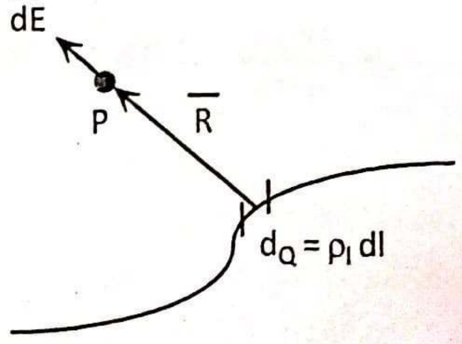
$$Q = \int_L \rho_l \, dl$$

- Each differential charge dQ along the line produces a differential electric field at P

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_l \, dl}{4\pi\epsilon R^2}\,\bar{a}_R$$

- The total field at P is

$$\bar{E} = \int d\bar{E} = \int_L \frac{\rho_l \, dl}{4\pi\epsilon R^2}\,\bar{a}_R$$



(3) Sheet charge distribution "Surface"

- The charge is distributed over a surface with a surface charge density ρ_s (C/m^2)
- The total charge on the surface

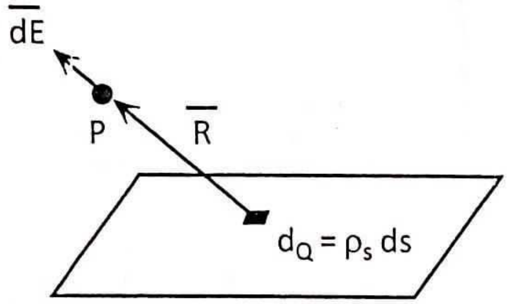
$$Q = \int_S \rho_s \, ds$$

- Each differential charge dQ along the surface results in a differential electric field

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2} \bar{a}_R = \frac{\rho_s \, ds}{4\pi\epsilon R^2} \bar{a}_R$$

- The total field at P is

$$\bar{E} = \int d\bar{E} = \int_S \frac{\rho_s \, ds}{4\pi\epsilon R^2} \bar{a}_R$$



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(4) Volume charge distribution "Surface"

- The charge is distributed through a specified volume with a volume charge density ρ_v (C/m^3)
- The total charge on the volume

$$Q = \int_V \rho_v \, dv$$

- Each differential charge dQ along the surface results in a differential electric field

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2} \bar{a}_R = \frac{\rho_v \, dv}{4\pi\epsilon R^2} \bar{a}_R$$

- The total field at P is

$$\bar{E} = \int d\bar{E} = \int_V \frac{\rho_v \, dv}{4\pi\epsilon R^2} \bar{a}_R$$

3.4 Standard Charge configurations

(1) Infinite Line charge

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_l\,dl}{4\pi\epsilon R^2}\,\bar{a}_R$$

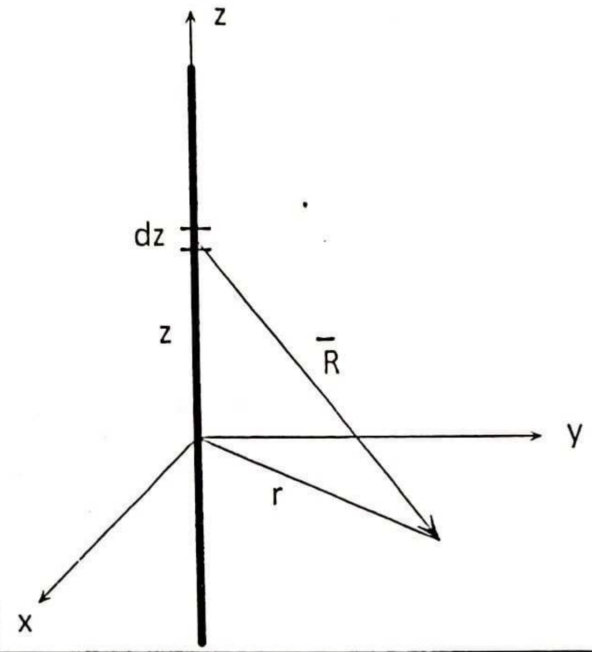
The total field at P is

$$\bar{E} = \int d\bar{E} = \int_L \frac{\rho_l\,dl}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_l}{4\pi\epsilon}\int_L \frac{dl}{R^2}\,\bar{a}_R$$

$$\bar{R} = z(-\bar{a}_z) + r(\bar{a}_r)$$

$$R = |\bar{R}| = \sqrt{z^2 + r^2}$$

$$\bar{a}_R = \frac{z(-\bar{a}_z) + r(\bar{a}_r)}{\sqrt{z^2 + r^2}}$$



$$\bar{E} = \frac{\rho_l}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{dz}{(z^2 + r^2)}\left[\frac{r\,\bar{a}_r + z\,(-\bar{a}_z)}{\sqrt{z^2 + r^2}}\right]$$

Due to line symmetry , There is a radial component only

$$\bar{E} = \frac{\rho_l}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{r\,dz\,\bar{a}_r}{(z^2 + r^2)^{\frac{3}{2}}}$$

$$\bar{E} = \frac{\rho_l\,r}{4\pi\epsilon}\left[\frac{z}{r^2\sqrt{z^2 + r^2}}\right]_{-\infty}^{\infty}\bar{a}_r$$

$$\bar{E} = \frac{\rho_l}{4\pi\epsilon\,r}\,[1 - (-1)]\bar{a}_r$$

$$\boxed{\bar{E} = \frac{\rho_l}{2\pi\epsilon\,r}\,\bar{a}_r}$$

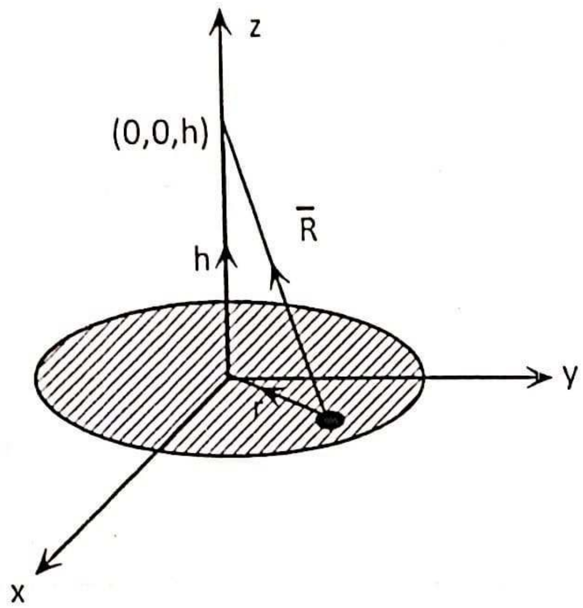
$\bar{E}$: المجال الكهربى عند نقطة بسبب خط طوله ∞ مشحون بـ ρ_l

r : البعد العمودى بين الخط و النقطة

$\bar{a}_r$: متجه وحدة من الخط للنقطة

(2) Infinite plane charge

- Electric field due to circular disk of radius a charged uniformly with surface charge density ρ_s



- **For infinite sheet**

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\,\bar{a}_z}$$

Generally

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\,\bar{a}_n}$$

$\bar{a}_n$: متجه وحدة عمودى من المستوى للنقطة

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7 A plane $y = 3$m contains a uniform charge distribution of a density $\rho_s = \left(\frac{10^{-8}}{6\pi}\right)$ C/m^2

Determine $\bar{E}$ at all points

Answer

$$\bar{E} = \frac{\rho_s}{2\epsilon}\,\bar{a}_n$$

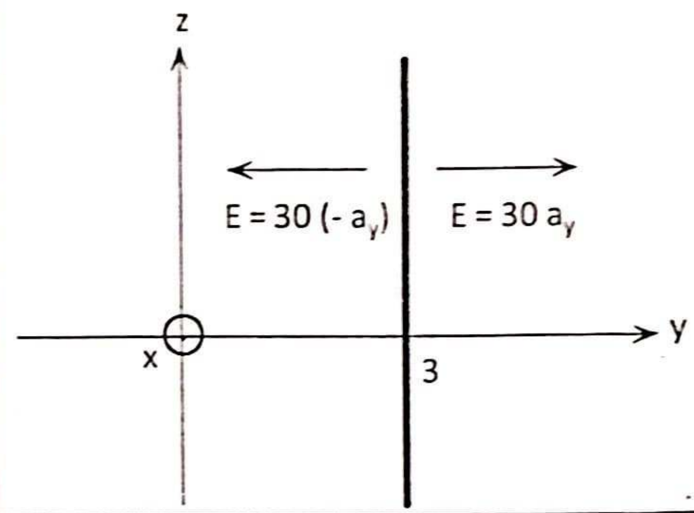
$$\frac{\rho_s}{2\epsilon} = \frac{\frac{10^{-8}}{6\pi}}{8.85 * 10^{-12}} \approx 30$$

for $(y > 3)$

$$\bar{E} = 30\,\bar{a}_y$$

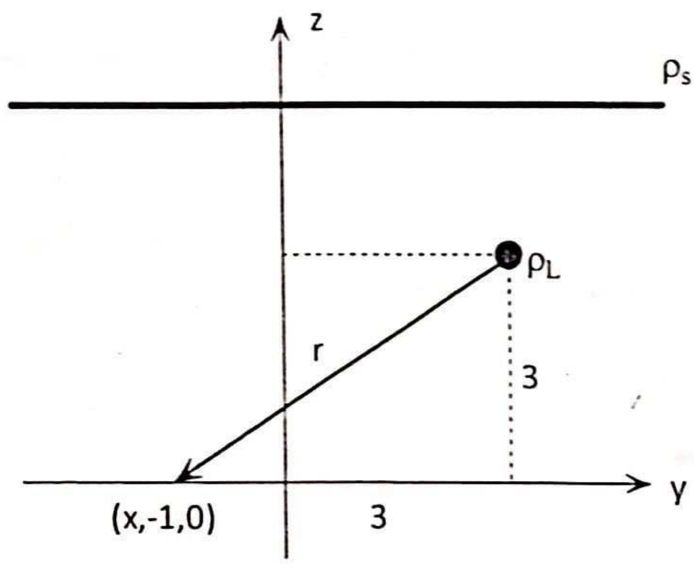
for $(y < 3)$

$$\bar{E} = -30\,\bar{a}_y$$



8 Determine $\bar{E}$ at $(x, -1{,}0)$m due to a uniform sheet charge with $\rho_s = \left(\frac{1}{3\pi}\right) nC/m^2$ is located at $Z = 5m$ and a uniform line charge with $\rho_l = \left(\frac{-25}{9}\right) nC/m$ at $z = +3$,$y = 3\ m$.

Answer



$$\bar{E} = \bar{E}_L + \bar{E}_S$$

$$\bar{E} = \frac{\rho_l}{2\pi\epsilon\, r}\,\bar{a}_r + \frac{\rho_S}{2\epsilon}\,\bar{a}_n$$

$$\bar{E} = \frac{-\frac{25}{9} \times 10^{-9}}{2\pi(8.85 \times 10^{-12})\,(5)}\left(\frac{-3\bar{a}_z - 4\bar{a}_y}{5}\right) + \frac{\frac{1}{3\pi} \times 10^{-9}}{2(8.85 \times 10^{-12})}(-\bar{a}_z)$$

$$\bar{E} = -8\bar{a}_y - 12\bar{a}_z$$